WARM-UP

Factor the expression.

\[63x - 28x^3\]
WARM-UP

Factor the expression.

$$24x^3 - 18x^2 - 15x$$
Solving Linear Equations with variables on both sides of the equation.

An equation is solved if the following are true.

1. The variable that you want to solve for is on one side of the equation.

2. The variable that you want to solve for has a coefficient of positive 1.

3. The variable that you want to solve for has an exponent of 1.
Steps to solving a linear equation.

1. If possible simplify each side of the equation removing grouping symbols.

2. If possible, combine like terms (MUST to be on the same side of the equation).

3. If the variable that you want to solve for is on both sides of the equation cancel out one of the variables.

4. Add (or subtract) the same real number or variable expression on both sides of the equation.

5. Multiply (or divide) by the same nonzero number on both sides of the equation.
EXAMPLE 1

Solve: \(-2x - 9 - x = 5 + 4x\)

SOLUTION

Place a 1 in front of any variable that does not have a coefficient.

\[-2x - 9 - 1x = 5 + 4x\]

\[-3x - 9 = 5 + 4x\]

\[-3x - 9 = 5 + 4x\]

\[-3x - 9 = 5 + 4x\]

\[+3x + 3x\]

\[-9 = 5 + 7x\]
\[-9 = 5 + 7x\]
\[-9 = 5 + 7x\]
\[-5 \quad -5\]
\[-14 = 7x\]
\[-\frac{14}{7} = \frac{7x}{7}\]
\[-2 = x\]
This time the equation will be solved cancelling out the $4x$.

\[-3x - 9 = 5 + 4x\]

\[-3x - 9 = 5 + 4x\]

\[-4x\]

\[-7x - 9 = 5\]
Now that the variable is only on 1 side of the equation. The last steps are to isolate the variable.

\[-7x - 9 = 5\]

\[-7x = 14\]

\[x = -2\]
EXAMPLE 2

Solve:

\[-7 - 3x - 1 + x = -1 + 6x - 4 - 5x\]

SOLUTION

Place a 1 in front of any variable that does not have a coefficient.

\[-7 - 3x - 1 + 1x = -1 + 6x - 4 - 5x\]

\[-8 - 2x = -5 + 1x\]

\[-8 - 2x = -5 + 1x\]

\[\text{+2x} \quad \text{+2x}\]

\[-8 = -5 + 3x\]
Now that the variable is only on 1 side of the equation. The last steps are to isolate the variable.

\[-8 = \boxed{-5} + \mathbf{3x}\]

\[-8 = -5 + 3x\]

\[+5 \quad +5\]

\[\underline{-3 = 3x}\]

\[-3 \quad = \quad 3x\]

\[\frac{-3}{3} = \frac{3x}{3}\]

\[\frac{-3}{3} = \frac{3x}{3}\]

\[-1 = x\]
EXAMPLE 3

Solve:

\[-4x - 3(2x + 5) = 2 - 8x - 5\]

SOLUTION

Simplify by distributing to eliminate the grouping symbols.

\[-4x - 3(2x + 5) = 2 - 8x - 5\]

\[-4x - 6x - 15 = 2 - 8x - 5\]

\[-10x - 15 = -3 - 8x\]

\[+8x\]

\[-2x - 15 = -3\]
\[\begin{align*}
-2x - 15 &= -3 \\
+15 &+15 \\
\hline
-2x &= 12
\end{align*}\]
\[\frac{-2x}{-2} = \frac{12}{-2}\]
\[x = -6\]
EXAMPLE 4

Solve:

\[ 3(2x - 1) - (7 - 4x) + 6 = 6x - 5(x + 2) + 9x + 6 \]

SOLUTION

Place a 1 in front of any variable that does not have a coefficient and in front of the parentheses.

\[
3(2x - 1) - 1(7 - 4x) + 6 = 6x - 5(1x + 2) + 9x + 6
\]

\[
6x - 3 - 7 + 4x + 6 = 6x - 5x - 10 + 9x + 6
\]

\[
10x - 4 = 10x - 4
\]
\[
10x - 4 = 10x - 4 \\
-10x \\
\underline{-10x} \\
0x - 4 = 0x - 4 \\
0 - 4 = 0 - 4 \\
-4 = -4
\]

The statement is **TRUE**, both sides of the equation are equal.

**ALL REAL NUMBERS**
EXAMPLE 5

Solve: \(-\frac{1}{2}x + \frac{3}{2} = \frac{7}{2} - \frac{5}{2}x\)

SOLUTION

The every denominator in the equation is 2 so the least common multiple is 2.

Multiply the entire equation by 2.

\[2\left(-\frac{1}{2}x + \frac{3}{2} = \frac{7}{2} - \frac{5}{2}x\right)\]
\[2 \cdot -\frac{1}{2}x + 2 \cdot \frac{3}{2} = 2 \cdot \frac{7}{2} - 2 \cdot \frac{5}{2}x\]

\[-1x + 3 = 7 - 5x\]

\[+1x\]

\[3 = 7 - 4x\]
\[
3 = 7 - 4x \\
\begin{array}{c}
-7 \\
-7 \\
\hline
-4 = -4x
\end{array}
\]

\[
-4 = -4x \\
\begin{array}{c}
-4 \\
-4 \\
\hline
-4 = -4x
\end{array}
\]

\[
1 = x
\]
EXAMPLE 6

Solve: \( \frac{5}{6}x - \frac{11}{2} = -\frac{8}{3}x + 7 - \frac{1}{4}x \)

SOLUTION

The denominators are the numbers 6, 2, 3 & 4 so the least common multiple is 12.

Multiply the entire equation by 12.

\[ 12 \left( \frac{5}{6}x - \frac{11}{2} = -\frac{8}{3}x + 7 - \frac{1}{4}x \right) \]
12 \cdot \frac{5}{6} x - 12 \cdot \frac{11}{2} = 12 \cdot \frac{11}{3} x + 12 \cdot 7 - 12 \cdot \frac{1}{4} x

\frac{2}{12} \cdot \frac{5}{6} x - \frac{6}{12} \cdot \frac{11}{2} = \frac{4}{12} \cdot \frac{11}{3} x + 12 \cdot 7 - 3 \cdot \frac{1}{4} x

2 \cdot 5x - 6 \cdot 11 = 4 \cdot 8x + 12 \cdot 7 - 3 \cdot 4x

10x - 66 = -32x + 84 - 12x

10x - 66 = -44x + 84

+44x +44x

54x - 66 = 84
\[ 54x - 66 = 84 \]
\[ + 66 + 66 \]
\[ \frac{54x}{54} = \frac{150}{54} \]
\[ x = \frac{150}{54} \]
\[ x = \frac{150 \div 6}{54 \div 6} \]
\[ x = \frac{25}{9} \]
EXAMPLE 7

Solve: \( \frac{x - 3}{4} = \frac{5}{14} - \frac{x + 5}{7} \)

SOLUTION

Place a 1 in front of any variable that does not have a coefficient.

\[ \frac{1x - 3}{4} = \frac{5}{14} - \frac{1x + 5}{7} \]

\[ 28 \left( \frac{1x - 3}{4} = \frac{5}{14} - \frac{1x + 5}{7} \right) \]

\[ 28 \cdot \frac{1x - 3}{4} = 28 \cdot \frac{5}{14} - 28 \cdot \frac{1x + 5}{7} \]
Solving Linear Equations with variables on both sides

\[
\frac{7}{28} \cdot \frac{1x - 3}{1} = \frac{2}{28} \cdot \frac{5}{14} - \frac{4}{28} \cdot \frac{1x + 5}{7}
\]

\[
\frac{7(1x - 3)}{1} = \frac{2 \cdot 5}{1} - \frac{4(1x + 5)}{1}
\]

\[
7(1x - 3) = 2 \cdot 5 - 4(1x + 5)
\]

\[
7(1x - 3) = 2 \cdot 5 - 4(1x + 5)
\]

\[
7x - 21 = 10 - 4x - 20
\]

\[
7x - 21 = -10 - 4x
\]
\[
\begin{align*}
7x - 21 &= -10 - 4x \\
+4x & \quad +4x \\
11x - 21 &= -10 \\
+21 & \quad +21 \\
11x &= 11 \\
\frac{11x}{11} &= \frac{11}{11} \\
\frac{11x}{11} &= \frac{11}{11} \\
x &= 1
\end{align*}
\]